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Estimation of Formation Hydraulic Properties from Analysis of Regular Operations Data

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Abstract

We propose to use regular pumping rate - pumping pressure data for estimating the formation hydraulic properties in the vicinity of wellbore without interrupting the operations. A distinctive feature of our analysis is that we account for the effects of pumping preceding the test interval. We derive our model from the same assumptions as in a standard pressure draw-down or pressure build-up test analysis. We introduce an additional parameter characterizing an effective pumping rate prior to the test. This parameter is of crucial importance for our procedure because there is no shut-in period preceding the test during regular operations. Moreover, we demonstrate that accounting for this parameter does affect analysis of a traditional pressure draw-down or build-up test as well. Our method produces good data matching, and the results are stable with respect to the selection of the portion of data for fitting. We derive our conclusions after analyzing almost entire data set, rather than only a portion of the pressure fall-off or build-up curve. This makes the results more reliable and stable with respect to measurement errors.

The new method we have developed is implemented in a code named ODA (Operational Data Analysis). This program incorporates a special curve-fitting procedure which we designed for our method to significantly simplify the problem and reduce the amount of computations. Therefore, the analysis can be preformed on a laptop computer.

Introduction

Using a well test to estimate hydraulic formation properties in the vicinity of wellbore is a common practice in petroleum and environmental industries. The basic theory of well test analysis was proposed in an early work of Theis (Ref. 1) and was substantially enhanced in later research. The accrued results and experience are summarized in several monographs and surveys Refs. 2-8. The traditional technique usually requires interrupting regular operations for a certain period of time. During this time, special operations are performed at the well, and the pumping rates and pressures are measured and analyzed. Such operations normally include shutting-in the well and impose additional costs on the operator.

Instead of interrupting the operations, we propose to select a portion of pumping data over a certain time interval and analyze this information. Such a situation introduces new elements into analysis of the data. The injection rate, which is usually maintained constant in a traditional well test, can be arbitrary varying in time. Also, the absence of a shut-in period preceding the test implies that the models used for data interpretation may not work. The reason for this is that traditional methods use the solution to flow equation which is valid only if the impact of pre-test pumping can be neglected. To address this issues, in our previous papers, Refs. 9, 10, we performed rigorous analysis of error introduced by such an assumption. We obtained that the difficulties with interpreting well test data can be partially explained through this error. Moreover, we proposed a modified solution which substantially decreases, and in some cases practically eliminates the problem of matching the data. The crucial point of our argument in Refs. 9, 10 was the introduction of an additional parameter for evaluating an effective pre-test pumping rate on an indefinite time interval preceding the test. This parameter, along with more traditional coefficients of transmissivity and storativity and skin factor, was used for matching the data curve. On examples, we demonstrated that the recovered value of the effective rate approximates the actual pre-test pumping rate with a remarkable accuracy. It is important to note that when our method is applied to traditional well test data set, we obtain a better fitting on a larger time interval than traditional methods. Moreover, the results are stable with respect to variations of this time interval. Comparison of our conclusions with results obtained independently using traditional methods also showed that our estimates of the skin factor are much lower, whereas the transmissivityity coefficient is recovered at a higher value. Correct estimation of these parameters is crucial for planning waterflood.

In this paper, we consider an alternative approach to account for pre-test pumping in well test data interpretation. The idea is as follows. From the point of view of the mathematical model of the flow into the formation, the principle consequence of pre-test pumping is non-uniform initial pressure distribution near the wellbore at the beginning of the test. If the pumping prior to the test was performed at approximately constant rate, then it is natural to assume that the initial pressure distribution is steady-state. Such an idea was explored in Refs. 3, 11. We propose to use the injection rate corresponding to this steady-state solution as an additional fitting parameter. We demonstrate that we obtain the same modified equation as we developed earlier in Refs. 9, 10. However, the error analysis we performed using indefinite time interval of pre-test pumping become unavailable on this way. At the same time, we obtain an easy way to evaluate an effective dimensionless wellbore radius. This radius may differ from the actual radius of tubing at perforation interval. There are several reasons for this difference. A test procedure and further analysis actually recover integral properties of the formation near the wellbore, whereas in fact these properties usually are heterogeneous. The formation near the wellbore is subject to damage caused by various chemical factors, see Ref. 12. In addition to that, the rock can be fissured, so the model of radial flow is approximate. The steady-state solution itself (see below) is expressed through a logarithmic function of the distance from the well, therefore, it is physically meaningful only within a certain length scale. Although the effective radius is a conventional parameter, comparison of results of well tests performed at different times can provide an important information about the character of changes in the formation.

An estimation algorithm based on our method has been implemented in a computer code named ODA (Operations Data Analysis). In example below, we use computations performed using this code.

The paper is organized as follows. First, we present a solution to a transient flow equation which is valid for a steady-state initial pressure distribution and variable injection rate. We derive an equation for estimating the formation parameters by fitting the measured pressure curve and compare it with the solution obtained in Refs. 9, 10. Second, we describe the special minimization procedure we designed for fitting the data curve. Third, we propose a method of estimating the ambient reservoir pressure and effective dimensionless wellbore radius using. The idea of the method is to simulate a special well test using the results of data fitting. Fourth, we present an example of application of our procedure. Finally, we formulate conclusions and describe step-by-step the procedure which we propose for estimating hydraulic formation properties using regular pumping data.

Theoretical Background

In this section we present the theoretical background of the method we propose.

The main assumption we use are the same as in a traditional well test analysis. Namely, we assume isothermal radial flow of a slightly compressible fluid into a homogeneous formation which is described by equation

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c}{k}\frac{\partial p}{\partial t} \qquad (1)$$

The initial and boundary conditions are given by:

$$p(r, t_0) = p_0(r) \qquad (2)$$

$$p(\infty, t) = p_\infty, \quad t \geq t_0 \text{ (ambient pressure at infinity)} \qquad (3)$$

$$\lim_{r \to +0} r\frac{\partial p(r,t)}{\partial r} = -\frac{\mu}{2\pi kH} Q(t) \qquad (4)$$

(a variable flow rate at the wellbore)

Here $p(t,r)$ is the fluid pressure at the time t and distance r from the well, ϕ is the porosity of the formation near the wellbore, μ is the viscosity of the fluid, k is the permeability, and c is the compressibility coefficient. In Equation (4), H is the thickness of the injection/production layer.

Equations (1)-(4) are well-known, see, e.g., Refs. 4, 13. If $p_0(r) \equiv p_\infty$, i.e. the initial pressure distribution is uniform, then the solution to problem (1)-(4) is given by equation

$$p(t,r) = p_\infty + A\int_{t_0}^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau} Q(\tau)d\tau \qquad (5)$$

where

$$A = \frac{\mu}{2\pi kH} \quad \text{and} \quad B(r) = \frac{\phi\mu c r^2}{4k} \qquad (6)$$

Coefficients A and B are introduced to simplify further calculations and are expressed through transmissivity T and storativity S by

$$A = \frac{1}{2\pi T} \qquad B = \frac{Sr^2}{4T} \qquad (7)$$

Conversely, transmissivity T and storativity S can be expressed through A and B:

$$T = \frac{1}{2\pi A} \qquad S = \frac{2B}{\pi r^2 A} \tag{8}$$

Unit conversion coefficients have to be introduced into Eqs (7) and (8) if the parameters are represneted in incompatible units. Solution (5) can be obtained from the well-known exponential integral solution through Duhamel integral, see Refs. 14, 15. If $Q(t) = Q_0 = Const$, then Eq. (5) implies that

$$p(t) = p_\infty - A\,\mathrm{Ei}\left(-\frac{B}{t-t_0}\right) \tag{9}$$

i.e. we obtain the exponential integral solution. However, the solution to problem (1)-(4) is not as simple in case where the initial pressure distribution $p_0(r)$ is not uniform.

In many practical situations the regular pumping rates do not vary too much. Thus, it is natural to assume that the initial pressure distribution corresponds to the steady-state flow established by the beginning of the test. The steady-state solution to Eq. (1) is provided by

$$p_0(r) = p_\infty + AQ_{-1}\ln\frac{r_\infty}{r} \tag{10}$$

where Q_{-1} is an effective pumping rate corresponding to the steady-state pressure distribution and r_∞ is the distance from the wellbore where the pressure is equal to the ambient pressure at infinity. Solution (10) is valid only for radii r within a certain range, because the logarithm on the right-hand side has an infinite limit both as $r \to 0$ and $r \to \infty$. In fact, r_∞ estimates the upper bound for the interval where the steady-state solution presented in Eq. (10) can be applied. If we substitute $p_0(r)$ from Eq. (10) as the initial condition into Eq. (2), then the solution to problem (1)-(4) is given by

$$p(t,r) = p_0(r) + A\int_{t_0}^{t}\frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}\left(Q(\tau)-Q_{-1}\right)d\tau \tag{11}$$

see e.g. Ref. 3. Simple calculations yield

$$p(t,r) = p_0(r) + A\int_{t_0}^{t}\frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)d\tau$$

$$+AQ_{-1}\,\mathrm{Ei}\left(-\frac{B}{t-t_0}\right) \tag{12}$$

If we put $r = r_w$, where r_w is an effective wellbore radius, then we obtain Eq. (1) from Ref. 9. A significant difference between the latter and Eq. (12) lies in the ways how these equations have been obtained. The derivation of Eq. (12) presented here actually hides some important estimates obtained in Ref. 10. At the same time, it suggests a way to estimate the effective wellbore radius, see below.

Assuming $r = r_w$ everywhere below, we omit the index w in further calculations without confusion.

To account for possible skin effect we introduce a dimensionless skin factor in a standard way, see Ref. 4, so that Eq. (12) transforms into

$$p(t) = p_0 + A\int_{t_0}^{t}\frac{\exp\left(-\frac{B}{t-\tau}\right)}{t-\tau}Q(\tau)d\tau$$

$$+AQ(t)\,s + AQ_{-1}\,\mathrm{Ei}\left(-\frac{B}{t-t_0}\right) \tag{13}$$

Thus, the wellbore pressure calculated at time t depends on five parameters: pumping pressure p_0 at the initial moment of time $t = t_0$, an effective pre-test pumping rate Q_{-1}, skin factor s and coefficients A, B defined in Eq. (6). Strictly speaking, the skin factor has to be added to Eq. (10) as well:

$$p_0(r) = p_\infty + AQ_{-1}\left(\ln\frac{r_\infty}{r} + s\right) \tag{14}$$

However, taking into account our remarks about the effective wellbore radius, we incorporate the skin factor into r_w,. so that for p_0 in Eq. (13) we get

$$p_0 = p_\infty + AQ_{-1}\ln\frac{r_\infty}{r_w} \tag{15}$$

The ratio r_w / r_∞ can be called a dimensionless effective wellbore radius. Changes of its magnitude reflect the formation damage near the well.

Now, with all calculations done, the data analysis reduces to matching the pressure curve with pressures calculated using formula (13).

Minimization procedure

In this section we describe a minimization procedure for matching the measured pressures.

The procedure goes the following way. First, we specify a time interval where we want to analyze the data. As it follows from estimates obtained earlier in Refs. 9, 10, it is preferable that the injection rate did not vary too much prior to this interval. Also, the flow during at least a part of the test time interval must be transient. The last requirement means that both pressures and pumping rates have to have substantial variation on this interval. Then, the entire selected time interval of pumping data is split into two parts. The earlier part can be called the beginning phase, the later part can be called the test phase. The test phase is used in a best-fitting procedure to estimate the formation parameters. Denote by t_0 and t_2, respectively, the beginning and the end of the whole selected interval, and denote by t_1 the splitting point between the beginning and test phases, see Fig. 1. Then, mathematically, the problem reduces to minimization of criterion

$$J = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} w_P(t)\big(p(t) - p_*(t)\big)^2 dt \qquad (16)$$

with respect to five parameters A, B, p_0, Q_{-1} and s. The integral criterion (16) can be replaced with a discrete one

$$J_N = \frac{1}{N} \sum_{i=1}^{N} w_i\big(p(\theta_i) - p_*(\theta_i)\big)^2 \qquad (17)$$

All weight coefficients w_i in Eq. (17) and function w_P in Eq. (16) are positive . Although there is no ultimate criterion for the best selection of those, they should take relatively large values at the points where the measured data is less affected by possible measurement error and side effects.

The further argument can be applied in either case in a similar way. For definiteness, we will focus here on integral criterion (16). The problem of minimization of criterion (16) with respect to parameters A, B, p_0, Q_{-1} and s is a nonlinear optimization problem. However, with change of variables

$$Z_1 = p_0, \qquad Z_2 = A \cdot Q_{-1}, \qquad Z_3 = A \quad Z_4 = A \cdot s \quad (18)$$

it is dramatically simplified. Indeed, the criterion (16) takes on the following form:

$$J = \frac{1}{2} \int_{t_1}^{t_2} w_P(t)\big(Z_1 + g(B;t)Z_2 + \varphi(B;t)Z_3\, dt$$

$$+ \psi(B;t)Z_4 - p_*(t)\big)^2 dt \qquad (19)$$

where

$$g(B;t) = \mathrm{Ei}\left(-\frac{B}{t - t_0}\right) \qquad (20)$$

$$\varphi(B;t) = \int_{t_0}^{t} \frac{\exp\left(-\frac{B}{t-\tau}\right)}{t - \tau} Q(\tau)\, d\tau \qquad (21)$$

and

$$\psi(B;t) = Q(t) \qquad (22)$$

Hence, for a fixed B, the functional (19) is quadratic with respect to the new variables Z_1, Z_2, Z_3, and Z_4. Consequently, its minimum can be explicitly calculated by equating to zero the derivatives of the functional J with respect to Z_1, Z_2, Z_3, and Z_4. Thus, we obtain a system of linear equations of fourth order, whose analytical solution reduces to inversion of a 4×4 matrix. From this analytical solution, we obtain

$$A(B) = Z_3(B), \qquad p_0(B) = Z_1(B), \qquad (23)$$

$$Q_{-1}(B) = Z_2(B)/Z_3(B)\cdot, \quad \text{and} \quad s(B) = Z_4(B)/Z_3(B) \quad (24)$$

Substituting Eqs. (23) and (24) back into (16), we end up with a function of only one variable

$$J(B) = J\big(B, A(B), p_0(B), Q_{-1}(B), s(B)\big) \qquad (25)$$

For minimization of function (25), a one-variable minimum search procedure can be applied. For example, in our code ODA we successfully applied the method of golden sections, see Refs. 16, 17.

The advantage of the procedure described above is that it avoids numerical minimization except minimization of a function of one variable. Therefore, our calculation are not as cumbersome and we avoid application of an iterative descent methods to a stiff multivariable optimization problem.

From Eq. (15) we infer that

$$\frac{r_w}{r_\infty} = \exp\left(-\frac{p_0 - p_\infty}{AQ_{-1}} \right) \tag{26}$$

Thus, to determine the dimensionless effective wellbore radius magnitude, we need information about the ambient pressure p_∞. Traditional methods, like Horner plot analysis, (e.g., Ref. 18) are based on analysis of data of a specially designed test, whereas in our case we deal with regular pumping data. However, provided the estimates of p_0, A and Q_{-1} are available, a well test for Horner plot analysis can be simulated and p_∞ can be recovered from analysis of this simulated data.

Estimating ambient reservoir pressure

The main idea of estimating the ambient pressure is to use Eq. (13) along with the estimated values of parameters p_0, A and Q_{-1} in order to produce a simulated well test data. Then these data can be analyzed using Horner plot analysis and the ambient reservoir pressure can be recovered from there. The advantage of the proposed method is that we can assume arbitrary injection rates and time intervals for simulating a well test. In fact, we can simulate a series of tests and take an average result to minimize the impact of computational errors.

The important assumption behind Eq. (13) is that the test period $[t_0, t_2]$ is preceded with a sufficiently long time period where the effective pumping rate was Q_{-1}. "Sufficiently long" time means that the pumping rate before the time moment t_0 does not affect too much the pressures on the interval $[t_1, t_2]$. Thus, in the simulations we can assume that Q_{-1} was the pumping rate all the way from the very beginning of operation at time $t = 0$. Also, the entire test period $[t_0, t_2]$ is much shorter than the time of operations,

$$t_0 \gg t_2 - t_0 \tag{27}$$

For simulations, we select a constant pumping rate Q_1 between t_0 and t_1, and zero pumping rate (shut-in) between t_1 and t_2. Then, from Eq. (5),

$$p(t) = p_\infty + AQ_{-1}\left[\mathrm{Ei}\left(-\frac{B}{t-t_0} \right) - \mathrm{Ei}\left(-\frac{B}{t} \right) \right]$$

$$+AQ_1\left[\mathrm{Ei}\left(-\frac{B}{t-t_1} \right) - \mathrm{Ei}\left(-\frac{B}{t-t_0} \right) \right] \tag{28}$$

For sufficiently large times we can approximate the exponential integrals on the right-hand side of Eq. (28) with natural logarithms, so that for $t_1 < t \le t_2$ we get

$$p(t) = p_\infty + AQ_{-1}\ln\left(\frac{t-t_0}{t} \right) + AQ_1 \ln\left(\frac{t-t_1}{t-t_0} \right) \tag{29}$$

From inequality (27), the argument of the first logarithm on the right-hand side of Eq. (29) is much closer to unity than that of the second logarithm. Thus, approximately,

$$p(t) \approx p_\infty + AQ_1 \ln\left(\frac{t-t_0}{t-t_1} \right) \tag{30}$$

Eq. (30) is essential for Horner plot analysis. Particularly, in our case, it implies that the plot of pressures versus

$$\eta = AQ_1 \ln\left(\frac{t-t_0}{t-t_1} \right) \tag{31}$$

is a straight line with the slope of 45 degrees. From Eq. (30), the ordinate of the cross-section of this line towards with $\eta = 0$ in the plane (η, p) produces an estimate for p_∞. Note that at a very large *t* both logarithms on the right-hand side of Eq. (29) become comparable and approximated Eq. (30) becomes invalid.

Now, let us turn back to Eq. (13): it is valid for sufficiently large times, so that estimate (27) holds true. At the same time, $t_2 - t_0$ should not be too large, as the exponential integral on the right-hand side of Eq. (13) goes to infinity as its argument tends to zero. For the simulated test we obtain from Eq. (13):

$$p(t) = p(t_0) + AQ_{-1}\,\mathrm{Ei}\left(-\frac{B}{t-t_0} \right)$$

$$+AQ_1\left[\mathrm{Ei}\left(-\frac{B}{t-t_1} \right) - \mathrm{Ei}\left(-\frac{B}{t-t_0} \right) \right] \tag{32}$$

Thus, by plotting pressures calculated using Eq. (32) versus variable η defined in Eq. (31), we must obtain a straight line with the slope of 45 degrees. Thus, to estimate p_∞, it only remains to find the intersection of this line with the vertical axis in the plane η, p.

To summarize, we have the following procedure of estimating p_∞.

(1) after estimating p_0 and Q_{-1} using minimization procedure from the previous section, select injection rate

Q_1 and times t_1 and t_2 to generate a pressure curve using Eq. (32).

(2) Plot this curve versus $\eta = AQ_1 \ln\left(\frac{t-t_0}{t-t_1}\right)$ and localize the part which is a straight line at the slope of 45 degrees.

(3) Fund the intersection of this straight line with the *p*-axis. The *p*-coordinate of this intersection provides an estimate for p_∞.

(4) Substitute the estimates obtained in steps (1)-(3) into Eq. (26) to calculate the effective dimensionless wellbore radius.

As the steps (1)-(2)-(3)-(4) require only simple calculations, they can be performed several times at different values of Q_{-1}, t_0, t_1 and t_2. For the final estimate we can take a mean value of the individual results.

Field example

In this section we analyze injection well test data using the methods proposed above. Minimization of fitting criterion (16) was performed with our code ODA (Operations Data Analysis).

The measurements were performed at the following conditions. No information about the operations prior to the test is available. The data set begins with a short shut-in time interval of about 4.5 hours. Then the injection was conducted at an approximately constant rate of 66 gallons per minute for approximately 90 hours with a short break. After that the well was shut in for approximately 100 hours. The injection rates and pressures were measured approximately every minute. The respective curves are presented in **Fig. 2** and **Fig. 3**. For our analysis, we considered only data points accrued hourly, so we restricted ourselves to a data file 60 times smaller than the original one. We obtained stable high quality data matching for various selections of t_0, t_1 and t_2. An example is presented in **Fig. 4:** the difference between measured and calculated pressures is very small and hardly visible on the plot. The difference between the data curves in Fig. 3 and Fig. 4 is because in Fig. 3 we plot the entire data set, whereas in Fig. 4 we have only analyzed data, i.e. 60 times less data points. In calculations presented in Fig. 4 we had $t_0 \approx 72$ hours, $t_1 \approx 90$ hours and $t_2 \approx 161$ hours, where the time was measured with respect to the beginning of entire original data interval. For this case, the results of estimation of coefficients A, and B, the effective pre-test injection rate and skin factor are presented in the first row of Table 1. The other two rows show results for other selections of t_0, t_1 and t_2. In all three cases we had similar quality of fitting. Note that the recovered parameters have very close values for all three runs as well. The actual injection rate before t_0 was fluctuating between 65 and 66.5 gpm with a short-time drop, as can be seen in Fig. 2. Thus, this parameter also has been recovered with high accuracy. The skin factor was estimated at a remarkably low value, whereas analysis of the same data set performed independently with traditional methods produced a skin factor two orders of magnitude larger than ours.

The actual time intervals which we analyzed include not only the fall-off portion of the pressure curve, but also some part where active injection was performed. Although the data points beyond t_2 were not used in the minimization procedure, the quality of fitting is preserved when the calculated curve is extended beyond t_2. Note, that for a Horner plot analysis usually only a small portion of the data at later times is selected.

Now let us proceed with estimation of the ambient reservoir pressure. Based on results in the first row of Table 1, we simulated a well test assigning $t_1 - t_0 = 160$ hours and $t_2 - t_1 = 250$ hours. By plotting the simulated pressure curve using Eq. (32) versus variable defined in Eq. (31), we achieve practically straight line inclined at 45 degrees, see **Fig. 5.** The deviation from the straight line at small η is explained by indefinite increasing of the exponential integral as its argument tend to zero and invalidity of approximation (30) at very large timers, as discussed above. Averaging the calculated ambient pressure obtained at different injection rates Q_1 we obtain p_∞=1131.8 psi for the results from the first line of Table 1. By applying this procedure to results from Table 1 we obtain estimates gathered in Table 2. Again, the estimates are very close to each other for all three runs.

Conclusions

A method to estimate formation hydraulic properties from regular operation data is proposed.

Our analysis of the impact of pre-test pumping on well test analysis which we started in earlier papers Refs. 9, 10 is enhanced and extended. In this paper, the effects of pre-test pumping are accounted for by assuming steady-state pressure distribution at the beginning of the test. We demonstrated that the main equation obtained on this way is the same as in Refs. 9, 10. The steady-state initial pressure distribution approach, however, hides important estimates of the impact of the pre-test pumping which we obtained in the earlier papers. At the same time, it allows us to estimate another important parameter: an effective dimensionless wellbore radius.

We developed a minimization procedure for estimating the reservoir transmissvity and storativity along with skin factor which combines analytical calculations with numerical minimization of a function of one variable. The procedure proved to be efficient and produced high-quality data matching in examples.

We enhanced our earlier analysis with a procedure of estimating ambient reservoir pressure and dimensionless wellbore radius. For this procedure a well test is simulated using estimated parameters obtained using the optimization procedure mentioned above. As in this part of our analysis we use only simulated data, no special operations at the well are required.

To summarize, the following procedure is proposed for estimating hydraulic formation properties from regular operations data.

(A) Select a time interval where pumping rate and pumping pressure measurements are available. The flow has to demonstrate transient character at least in a part of selected interval.

(B) Split the entire selected time interval into two parts. The earlier part can be called the beginning phase, where the injection rate is only measured. The second part is used in a best-fitting procedure to estimate formation parameters.

(C) Apply the minimization algorithm described above to estimate the transmissivity, storativity, pre-test injection rate and skin factor. The algorithm is implemented into the code ODA (Operations Data Analysis).

(D) After the fitting procedure has been applied, use the results to estimate the ambient reservoir pressure and effective dimensionless wellbore radius. More specifically, follow steps (1)-(4) described above to simulate and analyze a set of well test data.

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Table 1. Results of the estimation procedure

t_0 [hrs]	t_1 [hrs]	t_2 [hrs]	A [psi/gpm]	B [day]	Q_{-1} [gpm]	s
72	90	160	0.89	0.006	67.83	-0.006
60	80	161	0.87	0.005	64.37	-0.074
50	80	170	0.87	0.005	64.39	-0.066

Table 2. Estimation of the ambient reservoir pressure and effective dimensionless wellbore radius

A [psi/gpm]	B [day]	Q_{-1} [gpm]	p_∞ [psi]	$\frac{r_w}{r_\infty}$
0.89	0.006	67.83	1131.8	0.0014
0.87	0.005	64.37	1146.0	0.0012
0.87	0.005	64.39	1141.7	0.0012

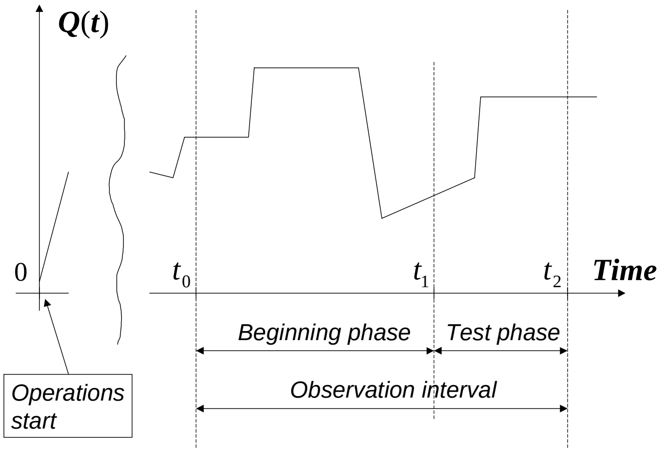

Fig. 1—The data interval schematic

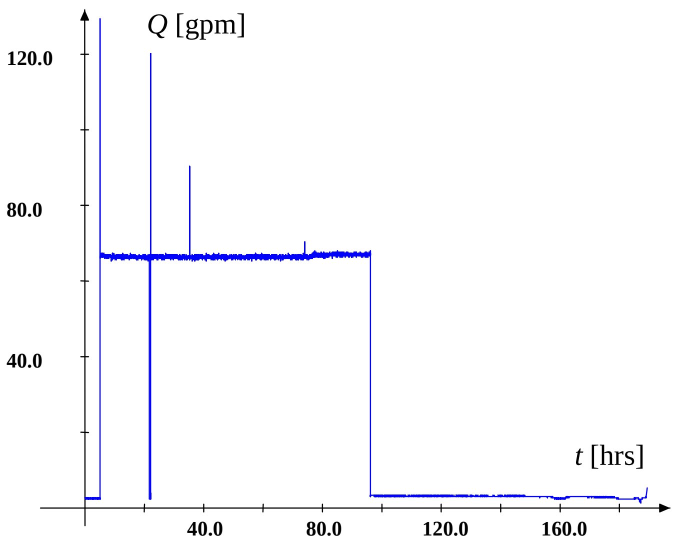

Fig. 2—Data: the injection rates

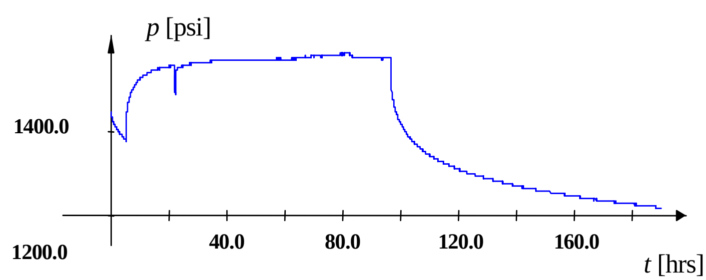

Fig. 3—Data: the injection pressures

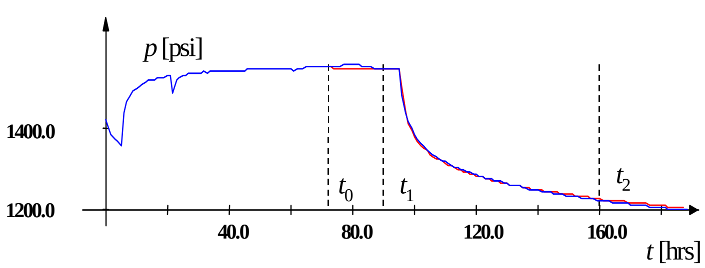

Fig. 4—The pressure curve fitting

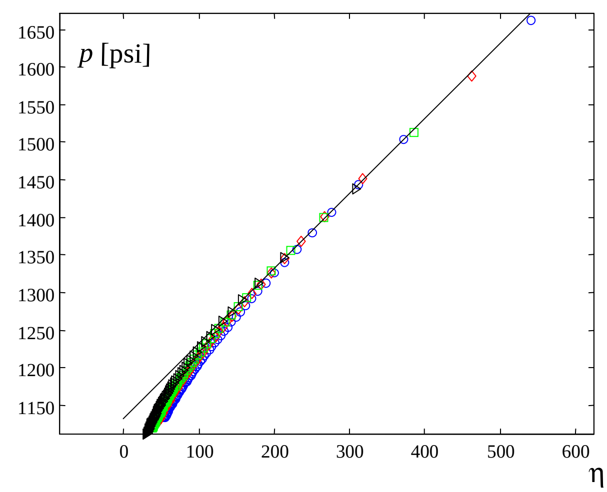

Fig. 5—Estimation of ambient reservoir pressure: circles - $Q_1 = 1.75Q_{-1}$**, diamonds -** $Q_1 = 1.5Q_{-1}$**, squares -** $Q_1 = 1.25Q_{-1}$ **and triangles -** $Q_1 = Q_{-1}$